

Open M5-branes

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We show how, in heterotic M-theory, an M5-brane in the 11-dimensional bulk may end on an “M9-brane” boundary, the M5-brane boundary being a Yang monopole 4-brane. This possibility suggests various novel 5-brane configurations of heterotic M-theory, in particular a static M5-brane suspended between the two M9-brane boundaries, for which we find the asymptotic heterotic supergravity solution.

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At the semi-classical level, M-theory is a theory of 11-dimensional supergravity interacting with its 1/2 supersymmetric branes, such as M2-branes and M5-branes, but also ‘M9-branes’ [1], which are actually boundaries that support a 10-dimensional supersymmetric E_8 gauge theory. Central to many applications of M-theory is an understanding of how some M-branes may have boundaries on others. This is in part due to the implications for superstring theories when viewed as M-theory compactifications. For example, the possibility of D-branes in type II string theory [2] can be deduced from the fact that M2-branes may have boundaries on M5-branes [3, 4]. Another example is the interpretation of the $E_8 \times E_8$ heterotic string as an M2-brane suspended between two M9-branes [1].

There is a further possibility for open branes that was recently emphasized by Polchinski in the context of heterotic string theories [5]. He has shown that the Green-Schwarz anomaly cancellation term implies that the $SO(32)$ heterotic string can have an endpoint on a ‘monopole’, defined as a point such that the integral of the 8-form $\text{Tr} F^4$ over an 8-sphere enclosing it is non-zero, where F is the Yang-Mills (YM) field-strength 2-form. This possibility is ‘non-standard’ because the ‘monopole’ has infinite energy, which means that the string endpoint is not free to move in the 9-dimensional space; the string can “end but not break”. Analogous “non-standard” open D-branes have recently been discussed [6].

Here we show that M5-branes may have boundaries on M9-branes. The boundary 4-brane has infinite tension, so its centre of mass is not free to move, although it may fluctuate. For a planar boundary 4-brane one can ignore the four directions of the brane; from this perspective the M5-brane boundary is a Yang monopole [7], which can be defined generally as a singular point of $SU(2)$ YM fields in 5-space (ignoring time) for which the integral of $\text{Tr} F^2$ over any 4-sphere enclosing it is non-zero (with $SU(2) \subset E_8$ here). This possibility allows several novel 5-brane configurations, such as the suspension of an M5-brane between two M9-branes, for which we find

the asymptotic form of the corresponding heterotic supergravity solution.

We begin with the bosonic truncation of the effective heterotic supergravity theory, without the quantum anomaly cancellation term. This involves the 10-dimensional supergravity fields (metric g , dilaton ϕ , and 2-form potential B with 3-form field strength H) and the Yang-Mills gauge fields. If all fermion fields are omitted, the Lagrangian density is

$$\mathcal{L} = \sqrt{-g} e^{-2\phi} \left[R + 4|\partial\phi|^2 - \frac{1}{2}|H|^2 - \kappa \text{Tr}|F|^2 \right], \quad (1)$$

where the trace is taken in the adjoint representation, and κ is a constant proportional to the inverse string tension. The 3-form field strength for B is $H = dB + \kappa\omega$, where ω is the Chern-Simons 3-form satisfying $d\omega = \text{Tr} F^2$, as a consequence of which H satisfies the ‘anomalous’ Bianchi identity

$$dH = \kappa \text{Tr} F^2. \quad (2)$$

A heterotic string is an electric source for B with a string charge $Q_1 \propto \int e^{-2\phi} \star H$, where the 7-form $\star H$ is the Hodge dual of H , and the integral is over any 7-sphere threaded by the string. Because the equations of motion imply that $e^{-2\phi} \star H$ is a closed form, this 7-sphere can be deformed arbitrarily, without changing the value of the integral, as long as no singularities of H are crossed. The presence of a heterotic string implies a singularity of H at the string core, but if the string were to have an endpoint we could contract the 7-sphere to a point without crossing this singularity, and thereby deduce that $Q_1 = 0$. Thus free heterotic string endpoints are forbidden, classically, although new possibilities can arise as a consequence of anomaly cancellation terms [5].

In addition to strings, heterotic string theories also have 5-branes. A planar 5-brane carries a 5-brane charge

$$Q_5 = \frac{1}{32\pi^2\kappa} \int_{S^3_\infty} H, \quad (3)$$

where the integral is over the 3-sphere at transverse spatial infinity. There are two possible contributions to this integral, corresponding to two types of 5-brane. For the ‘solitonic’ 5-brane [8], which we will call the H5-brane, the 3-sphere at infinity can be contracted to a point on a transverse 4-space Σ on which all fields are non-singular. In this case, the Bianchi identity (2) yields

$$Q_5 = \frac{1}{32\pi^2} \int_{\Sigma} \text{Tr} F^2. \quad (4)$$

We shall focus on the $E_8 \times E_8$ theory and assume that the YM fields of the H5-brane take values in the Lie algebra of the $SU(2)$ factor of an $E_7 \times SU(2)$ subgroup of one of the E_8 factors of $E_8 \times E_8$. In this case Q_5 is the $SU(2)$ instanton number. The worldvolume dynamics of a one-instanton H5-brane is governed by an action involving 30 worldvolume hypermultiplets [9].

The other type of 5-brane [10] can be found by shrinking the H5 instanton to zero size. In this process the region in which the YM fields are non-zero shrinks to a point, but this point simultaneously recedes to infinite affine distance as an infinite ‘throat’ forms in which the spacetime becomes the product of a 3-sphere of fixed radius and a 7-dimensional ‘linear-dilaton’ vacuum [9, 11]. In this limit, the YM fields become gauge-equivalent to the zero-field configuration, leaving a purely gravitational ‘black’ 5-brane. However, a ‘large’ gauge transformation is needed to gauge away the YM fields and this requires a transformation of B that transforms H into a closed but non-exact 3-form with a non-zero integral over the 3-sphere, such that the charge Q_5 remains the same. The linearly increasing dilaton implies a strong coupling limit, indicating that this ‘black’ 5-brane is best understood from the M-theory perspective. In fact, it is just the M5-brane interpreted as a solution of the minimal 10-dimensional supergravity [12], and the above process in which the H5-brane is converted into a ‘black’ 5-brane can be interpreted as one in which a 5-brane is pulled from the 10-dimensional boundary into the 11-dimensional bulk [13].

Now imagine that the above process, converting an H5-brane into an M5-brane, is carried out not in time but in some space direction, so that the instanton core of a planar H5-brane goes to zero size everywhere on a 4-plane. Since the 4 planar directions play no role we may periodically identify in these directions and consider the same process for a string in an effective 6-dimensional heterotic supergravity theory. Omitting the supergravity fields we have an instanton string in which the core shrinks to zero size at some point, resulting in a Yang monopole endpoint. To see this, consider the topological 4-sphere that results from the union of a 4-ball in Σ with a hemi-4-spherical ‘cap’ of the same radius R , in the limit of large R . As $F = 0$ on the cap, the integral of $\text{Tr} F^2$ over this 4-sphere reduces to the integral over Σ , which is $32\pi^2 Q_5$, so the instanton string ends on a Yang monopole of strength Q_5 . In the context of heterotic M-theory, the instanton string endpoint becomes the 4-

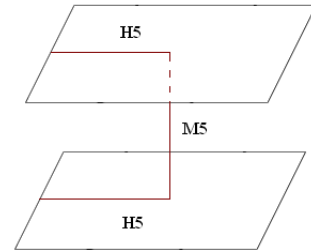


FIG. 1: An open 5-brane

dimensional interface between an M5-brane in the bulk and an H5-brane in the M9-brane boundary. In effect, the M5-brane boundary is a Yang-monopole 4-brane, but one with only $SO(4)$ symmetry rather than the $SO(5)$ symmetry of Yang’s original solution of the YM equations on \mathbb{E}^5 [7]. This is expected since no non-zero value of the 3-form H could be compatible with $SO(5)$ symmetry.

Let us now suppose that we have an M5-brane suspended across an interval bounded by two M9-branes, with a Yang monopole 4-brane boundary on each of the two M9-branes. From a 10-dimensional perspective, we now have *two* Yang monopoles, one in an $SU(2)$ subgroup of one E_8 and another in an $SU(2)$ subgroup of the other E_8 . Interpreting the Yang monopole as a semi-infinite H5-brane would allow two possible $SO(4)$ -invariant configurations, as shown in Figs. 1 and 2. Fig. 1 illustrates a configuration that has a 10-dimensional interpretation as a 5-brane ending on a ‘double’ Yang monopole of monopole ‘number’ $(1, -1)$, the first entry being the instanton number for the Yang monopole on one boundary and the second entry being the instanton number for the Yang monopole on the other boundary. The sign is determined by the contribution to the 5-brane charge Q_5 , which must vanish for this configuration. We thus have an open 5-brane of the heterotic string theory, which will be unstable against the formation of ‘holes’ with $(1, -1)$ Yang-monopole boundaries. Fig. 2 illustrates a configuration that has a 10-dimensional interpretation as a ‘kink’ on an H5-brane; on either side we have an H5-brane with $Q_5 = 1$, but on one side this arises from instanton ‘number’ $(1, 0)$ and on the other side from instanton ‘number’ $(0, 1)$. As the YM flux is ‘incoming’ for one H5-brane and ‘outgoing’ for the other one, the kink 4-brane is again a Yang monopole of monopole ‘number’ $(1, -1)$.

We noted above that no non-zero H is compatible with $SO(5)$ symmetry, but now that we have two Yang monopoles it is possible to arrange for H to vanish. From the Bianchi identity (2) we see that $H = 0$ requires

$$\text{Tr}_1 F^2 + \text{Tr}_2 F^2 = 0 \quad (5)$$

where the subscripts indicate a trace over the adjoint of

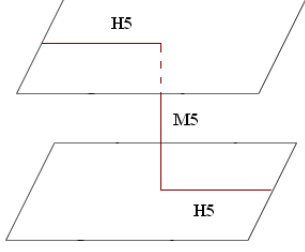


FIG. 2: A 5-brane ‘interface’

the Lie algebra of either the ‘first’ or the ‘second’ E_8 factor of the $E_8 \times E_8$ gauge group. This constraint is satisfied by coincident $SO(5)$ invariant Yang monopoles with monopole ‘number’ $(1, -1)$. Such a solution represents a static M5-brane suspended between two M9-branes, as shown in Fig. 3. The 4-brane boundary on each M9-brane is now a *spherically-symmetric* Yang monopole. Individually both are sources for H but their contributions cancel.

These considerations lead us to seek a static planar 4-brane solution of heterotic supergravity that is a source for a $(1, -1)$ Yang monopole. We can find this solution by a lift to 10 dimensions of a corresponding solution of the effective 6-dimensional theory obtained by toroidal compactification. Using the ansatz

$$ds_{10}^2 = e^{\sigma/2} ds_6^2 + e^{-\sigma/2} ds^2(T^4), \quad \phi = \sigma, \quad (6)$$

one finds a consistent truncation to 6-dimensional gravity, in Einstein frame, coupled to a 6-dimensional dilaton field σ , a 3-form field strength, and the $E_8 \times E_8$ YM fields. A further truncation of the 3-form field strength is inconsistent, in general, but for the special solution we seek it can be consistently set to zero. We may also consistently truncate each of the E_8 multiplets of YM fields to an $SU(2)$ triplet. The resulting 6-dimensional Lagrangian density is

$$\mathcal{L}_6 = \sqrt{-\det g} [R - (\partial\sigma)^2 - 4\kappa e^{-\sigma} \text{tr}|F|^2] \quad (7)$$

where the gauge group is now $SU(2) \times SU(2)$ and the trace is taken in the fundamental $(\mathbf{2}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2})$ representation; this explains the additional factor of 4. We now seek spherical symmetric solutions of this model that generalize the self-gravitating Yang monopole solutions of [14] to include the dilaton. Spherical symmetry implies that the metric takes the form

$$ds_6^2 = -e^{2\lambda(r)} \Delta(r) dt^2 + dr^2/\Delta(r) + r^2 d\Omega_4^2 \quad (8)$$

in terms of two functions λ and Δ , where $d\Omega_4^2$ is the $SO(5)$ invariant metric on the unit 4-sphere. It is convenient to set

$$\Delta(r) = 1 - 2\mu(r)/r^3 \quad (9)$$

for ‘mass function’ $\mu(r)$. In the absence of the σ field this model has a self-gravitating Yang monopole solution for each $SU(2)$ subgroup of the $SU(2) \times SU(2)$ gauge group. Because the YM field strength 2-form for this solution has components only on the 4-sphere, it continues to solve the YM equations as modified by the dilaton. One thus finds that

$$\text{tr}|F^2| = 2 \times 3/r^4 \quad (10)$$

where we have used the result of [14] for the $SU(2)$ self-gravitating Yang monopole and the factor of 2 arises from the necessity to consider a $(1, -1)$ Yang monopole. The Einstein and dilaton equations then reduce to the equations

$$\Delta r^4 \sigma'' + 12\kappa e^{-\sigma} + (4r^3 - 2\mu - 6\kappa e^{-\sigma} r) \sigma' = 0, \quad (11)$$

$$\mu' - 3\kappa e^{-\sigma} - \frac{1}{8}(\sigma')^2 r^4 \Delta = 0, \quad (12)$$

which are a pair of coupled ODEs for $\mu(r)$ and $\sigma(r)$, and the one further equation

$$4\lambda' = r(\sigma')^2, \quad (13)$$

which can be solved for $\lambda(r)$, given $\sigma(r)$, up to an irrelevant integration constant which we choose such that $\lambda(\infty) = 0$. It is useful to note that these three equations imply the σ equation of motion

$$(e^\lambda \Delta r^4 \sigma')' = -12\kappa e^{-\sigma} e^\lambda. \quad (14)$$

We have not found an explicit solution to these equations but one can find an asymptotic solution of the form

$$\sigma = \sigma_0 + \frac{A}{r^2} + \frac{\Sigma}{r^3} + \frac{A^2}{2r^4} + \dots \quad (15)$$

$$\mu = \frac{Ar}{2} + \mu_0 - \frac{A\Sigma}{2r^2} - \frac{4A^3 + 9\Sigma^2}{24r^3} + \dots \quad (16)$$

where σ_0 , μ_0 and Σ are arbitrary constants, and

$$A = 6\kappa e^{-\sigma_0}. \quad (17)$$

The asymptotic solution for λ is then found to be

$$\lambda = -\frac{A^2}{r^4} - \frac{3A\Sigma}{5r^5} + \dots \quad (18)$$

The integration constant σ_0 determines the string coupling constant $g_s = e^{\sigma_0}$ (since the zero mode of the 10-dimensional dilaton ϕ equals σ_0). The integration constant μ_0 can be identified as a ‘Schwarzschild’ mass (which may be negative because the total mass is infinite). The integration constant Σ can be identified as a scalar charge, which could also be negative since it is manifest from the expansion for σ that there is also a linearly divergent contribution to this charge proportional to κ . Note that it is consistent to set $\mu_0 = \Sigma = 0$ and in

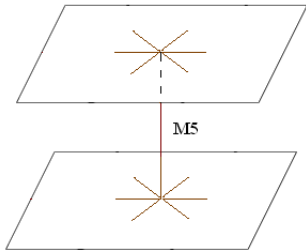


FIG. 3: An open M5-brane

this case the asymptotic expansion has the property that $\mu(r)$ is an odd function of r and $\sigma(r)$ an even function.

Having fixed the asymptotic behaviour by the choice of integration constants, the equations determine the behaviour in the interior. There cannot be a regular origin at $r = 0$ because regularity of Δ would imply $\mu'(0) = 0$ and then (12) requires $\sigma(0) = \infty$. There could be an event horizon, at $r = r_H > 0$ such that $\Delta(r_H) = 0$. In the absence of a Yang monopole (for which the equations are as above but with $\kappa = 0$) this is possible only if σ is constant, because then (14) implies that $e^\lambda \Delta r^4 \sigma'$ is a constant, which must vanish if all fields are regular as $\Delta \rightarrow 0$. Solutions of the Einstein-YM equations with a horizon exist in the presence of a Yang monopole [14], and an analysis of the conditions for a regular horizon in the present case shows that the integration constant Σ is determined in terms of A , so that σ_0 and μ_0 are the free parameters. This leads us to conjecture that there exist solutions of the Einstein-YM-dilaton equations considered here that are regular on and outside an event horizon. A detailed analysis would be needed to determine whether such a solution is stable, but we expect stability against splitting into spatially-separated (1, 0)

and (0, -1) Yang-monopoles because the interaction between them is entirely (super)gravitational. We also expect marginal stability against collapse to a configuration of the type shown in Figs. 1 and 2 because the energy of a spherically symmetric Yang monopole within a ball of radius r is $E(r) = Tr$, where T is precisely the instanton-string tension [14], which is the H5-brane tension in the current context.

Finally, we wish to point out that a membrane suspended between the two M9-branes could end on the M5-brane suspended between the two M9-branes. From the 10-dimensional perspective, this would appear to be an $E_8 \times E_8$ heterotic string with an endpoint on a (1, -1) Yang monopole. However, it is unclear to us what happens to the chiral modes of the string at this endpoint. Conceivably they leave the string in a manner that is analogous to that described by Polchinski for an endpoint of an $SO(32)$ heterotic string [5], but in that case the mechanism involved quantum anomaly considerations that are not obviously relevant here. We suspect that a proper understanding of open M5-branes and any M2-branes that end on them will involve quantum M-theory considerations.

In conclusion, we have provided a concrete realization of Yang monopoles in M-theory that may open up for investigation a new class of stable non-supersymmetric brane configurations in string theory. Previous discoveries of this nature have led to important insights into quantum field theories, and one may hope for similar insights from open M5-branes.

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